What to do when price changes are not normal

In theory, there is no reason that price changes cannot behave in a normally distributed way. But on some days, theories simply fly out of the window. At times like this, asks David Rowe, what can you do?

One of the great insights of modern statistical analysis is the Central Limit Theorem. It implies an amazingly pervasive set of circumstances that can be characterised statistically by the classical bell curve, or normal distribution. It is important to keep clearly in mind, however, what conditions give rise to this phenomenon. Essentially, the Central Limit Theorem applies when an observed value is the result of the impact of many relatively small independent factors.

The truly profound, and still rather amazing, insight is that these individual influences do not have to be normally distributed for their combined result to behave in this way. The individual influences can be distributed in almost any conceivable pattern. As long as they are statistically independent of each other, and their individual impacts are comparably weighted, the resulting variable will exhibit behaviour consistent with the normal distribution.

How does all this apply to daily changes in a market variable such as a foreign exchange rate or an interest rate? On most days, the application is quite direct. Market clearing prices are the result of decisions by large numbers of individuals (acting either on their own or on behalf of their organisations) to buy or sell in the marketplace. On most days, these decisions are the result of many independent factors brought to bear on different market participants.

Foreign trade receivables must be met, generating demand for foreign currency. A foreign subsidiary's dividends are repatriated, generating supply of foreign currency. A domestic obligation falls due and requires the liquidation of a fixed-income investment to meet the payment. A corporate treasurer gets worried about increasing interest rates and decides to liquidate long-term bonds and purchase short-term obligations. As long as the many factors influencing this plethora of individual decisions are largely independent, the resulting changes in market variables will be normally distributed.

The catch, of course, is that what is true for most days is not true of all days. One would have to go back to finance literature of the late 1960s to find any serious debate about whether market price changes are normally distributed. Empirical evidence clearly indicates that market-determined price changes are characterised by a class of leptokurtic distributions. These distributions combine bunching of small changes around the mean with an unusually frequent (although still rare) occurrence of very large changes.

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The most plausible explanation for consistently high kurtosis in observed market data is that the conditions for statistical normality do not always hold. This was represented recently in a scene from Rogue Trader, a film about Nick Leeson, whose activities brought down Barings Bank in 1995. In the film, Leeson has taken a large position that is vulnerable to a fall in the Nikkei index. He is awakened in the middle of the night by a phone call from a colleague. "You'd better turn on CNN," says the caller. As the TV screen flickers to life with scenes of the Kobe earthquake, the caller continues: "This is going to *kill* the market!" The caller's conclusion was correct and the rest is history.

There are, of course, many other recent examples of this phenomenon. Some are natural disasters such as the recent hurricanes and floods in the US or the earthquake in Taiwan. Others are the result of sudden decisions such as the withdrawal of the UK from the Exchange Rate Mechanism in 1992, the Mexican peso devaluation in 1995 or the Russian debt moratorium in 1998. Such events inject a pervasive new influence that changes the outlook of all market participants simultaneously.

In these circumstances, the model of a large number of unrelated (ie, statistically independent) and comparably weighted factors producing the change on the market price simply doesn't fit. A single dominant influence, with a clear directional implication, overwhelms the large number of small factors at work on such a day. This is sometimes referred to as the distinction between fluctuations and events.

Techniques

A variety of techniques have been applied to deal with the non-normality of price changes. One technique is to impose high kurtosis on the distributions used in Monte Carlo analysis. This can give a more realistic frequency of large changes in the results.

Other techniques involve jump diffusion processes or regime shifts. In effect, these treat price changes as resulting from random draws from two different distributions. The first is a normal distribution capturing daily fluctuations. This is modified by taking a draw from a zero/one distribution where the likelihood of drawing a one is the probability of a regime shift on any given day. On those (rare) days when a one is drawn, the price change based on the normal distribution is modified by a draw from another highly diffuse distribution. This has the advantage of being able to impose different correlations on the second distribution across variables, reflecting the observation that such correlations tend to be very different on "event" days than on days with normal fluctuations.

A second technique is to apply Extreme Value Theory as developed in the casualty insurance field. This effectively involves modelling the tails of the distribution (say the upper 5% and lower 5%) with one of a limited set of functions that experience indicates tend to track the pattern of extreme values.

These techniques have made a real contribution to our understanding of extreme events. I believe, however, that sound risk management requires us to supplement these approaches with less technical and more judgemental analysis. And that will be the subject of next month's column. ■